Spectral ocean model SOM

Reference Manual

Version 1.0

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1 Model equations

The spectral ocean model SOM is based on the Boussinesq equations for a baroclinic fluid. The current version does not resolve the ocean bathymetry and assumes a flat bottom. The model equations and the numerical solution procedure will be described in the following subsections.

1.1 Continuity equation

In agreement with the BOUSSINESQ approximation it is assumed that the threedimensional flow is nondivergent. In spherical coordinates the nondivergence condition reads

$$\frac{1}{r\cos\varphi}\frac{\partial u}{\partial\lambda}+\frac{1}{r\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi v)+\frac{\partial w}{\partial r}+\frac{2w}{r}=0\ . \tag{1.1}$$

In this equation λ, φ and r are the spherical coordinates longitude, latitude and radius, respectively and u, v and w denote the zonal, meridional and vertical velocity components of the velocity vector $\mathbf{v}$, respectively. The last term on the left hand side of this equation can be neglected since it is assumed that the ocean is contained within a thin spherical shell. Furthermore, the radius r can approximately be replaced by the mean planet radius a where it appears in a factor. Therefore, the continuity equation will be simplified to give

$$\frac{1}{a\cos\varphi}\frac{\partial u}{\partial\lambda}+\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi v)+\frac{\partial w}{\partial z}=\nabla_h\cdot\mathbf{v}_h+\frac{\partial w}{\partial z}=0, \tag{1.2}$$

where ∇_h denotes the horizontal LAPLACE operator, $\mathbf{v}_h$ the horizontal velocity vector and $z=r-a$ the height above the spherical surface having the radius $r=a$.

1.2 Momentum equation

The fluid of SOM is treated as a nondivergent viscous fluid. Therefore, the momentum equation is given by the BOUSSINESQ-approximated NAVIER STOKES' equation in a rotating coordinate frame:

$$\frac{\partial\mathbf{v}}{\partial t}+\nabla\left(\frac{1}{2}\mathbf{v}^2\right)+(\nabla\times\mathbf{v}+2\mathbf{\Omega})\times\mathbf{v}=-\frac{1}{\rho_0}\nabla p^*-g\frac{\rho^*}{\rho_0}\mathbf{k}+A_{mh}\nabla_h^2\mathbf{v}+A_{mv}\frac{\partial^2\mathbf{v}}{\partial z^2}, \tag{1.3}$$

where p^* is the pressure anomaly of a horizontally independent hydrostatic reference state of uniform density ρ_0, g the gravity acceleration in direction to the vertical unit vector $\mathbf{k}$, $\mathbf{\Omega}$ the angular velocity vector of the planet rotation which is directed perpendicular to the equatorial plane ($\varphi=0$), ρ^* the mass density anomaly, and A_{mh}

(A_{mv}) the horizontal (vertical) viscous momentum exchange coefficient. The index h indicates that only horizontal components or derivatives are considered.

SOM simulates flow phenomena having vertical scales that are much smaller than horizontal ones. In this case the hydrostatic balance is satisfied to a high degree. Therefore, it is assumed that the following hydrostatic balance equation holds

$$\frac{1}{\rho_0}\frac{\partial p^*}{\partial z} = -g\frac{\rho^*}{\rho_0} \ . \tag{1.4}$$

Furthermore, the REYNOLDS number of the flow in SOM takes a very small value since a large momentum exchange coefficient A_{mh} has to be chosen to properly parameterize the effect of mesoscale eddies that cannot be resolved by the model. Consequently, the nonlinear advection terms can be neglected and the horizontal momentum equation reads henceforth

$$\frac{\partial \mathbf{v}_h}{\partial t} + f\mathbf{k} \times \mathbf{v}_h = -\frac{1}{\rho_0}\nabla_h p^* + A_{mh}\nabla_h^2 \mathbf{v}_h + A_{mv}\frac{\partial^2 \mathbf{v}_h}{\partial z^2}, \tag{1.5}$$

where $f = 2\Omega \sin\varphi$ denotes the CORIOLIS parameter. The CORIOLIS force due to vertical motion has been neglected.

1.3 Equation of state

In the current model version density is only a linear function of temperature and salinity. Therefore, the equation of state simply becomes

$$\rho^* = \rho_0\left(-\alpha_T T^* + \alpha_S S^*\right) \ , \tag{1.6}$$

where T^* is the temperature anomaly, S^* the salinity anomaly, α_T the thermal expansion coefficient and α_S the salinity expansion coefficient. The anomalies refer to reference values T_0 and S_0.

An extension to include nonlinearities in the equation of state will be developed in the future.

1.4 Temperature and salinity budget equations

The temperature budget equation is based on the first law of thermodynamics and includes advection as well as heat conduction. The temperature equation becomes

$$\frac{\partial T^*}{\partial t} + \nabla_h \cdot (\mathbf{v}_h T^*) + \frac{\partial w T^*}{\partial z} = A_{Th}\nabla_h^2 T^* + A_{Tv}\frac{\partial^2 T^*}{\partial z^2} \ , \tag{1.7}$$

A_{Th} and (A_{Tv}) are the horizontal and vertical temperature exchange coefficients, respectively.

The budget equation for salinity takes the same form and is given by

$$\frac{\partial S^*}{\partial t} + \nabla_h \cdot (\mathbf{v}_h S^*) + \frac{\partial w S^*}{\partial z} = A_{Th} \nabla_h^2 S^* + A_{Tv} \frac{\partial^2 S^*}{\partial z^2} \ . \tag{1.8}$$

We assumed that the exchange coefficients for salinity are identical to those for temperature.

1.5 Boundary conditions

The current SOM model has a flat free-slip insulating bottom. The sea surface is movable where momentum and heat enters the ocean by air to sea fluxes. Furthermore, the kinematic boundary condition at the sea surface is linearized. Therefore, the boundary conditions at $z = -H$ (ocean bottom) and $z = 0$ (sea surface) take the form:

$$w = 0, \ \frac{\partial T^*}{\partial z} = 0, \ \frac{\partial \mathbf{v}_h}{\partial z} = 0 \qquad \text{at } z = -H \tag{1.9}$$

$$\frac{\partial h}{\partial t} = -\int_{-H}^{0} \nabla_h \cdot \mathbf{v}_h dz \ , \ A_{Tv} \frac{\partial T^*}{\partial z} = \frac{F_A}{c_w \rho_0} \ , \ A_{Tv} \frac{\partial S^*}{\partial z} = -(S^* + S_0) \frac{F_S}{\rho_0} \ , \tag{1.10}$$

$$A_{Mv} \frac{\partial \mathbf{v}_h}{\partial z} = -\frac{\boldsymbol{\tau}}{\rho_0} \ , \ p^* = g \rho_0 h \qquad \text{at } z = 0 \ .$$

where h denotes the height elevation of the sea surface, F_A the air to sea heat flux, c_w the heat capacity of sea water, F_S the fresh-water flux and $\boldsymbol{\tau}$ the wind stress.

1.6 Treatment of continents

Since the equations will be solved on the complete sphere with a spectral technique no boundary conditions can be incorporated at the coast lines. In fact, fluid also exists over continents, but the land exerts a drag as large as possible so that, practically, the flow remains zero over the continents. This can be realized by the introduction of the following force

$$\mathbf{F}_c = -\delta_{LS} \frac{\mathbf{v}}{\tau_c} \ , \tag{1.11}$$

where δ_{LS} is the land-sea mask function which becomes $\delta_{LS} = 1$ over land areas, and $\delta_{LS} = 0$ over oceanic areas. In order to obtain finite vorticity tendencies, the land-sea mask function can increase continuously from zero to one in a thin transition zone at the coast lines. In this manner, a large drag is introduced over continents by damping the flow with a continental friction time scale τ_C that should be much smaller than one time-step length. One may consider the force as a bottom friction force that becomes large near the coast lines and infinite over the continents due to the vanishing fluid depth.

1.7 Separation into barotropic and baroclinic part

For an efficient implicit numerical solution method it is useful to split the equations into a barotropic (vertically averaged) and a baroclinic (deviation) part. Vertical averaging (integration) of the momentum (continuity) equation leads to the barotropic part

$$\frac{\partial \hat{\mathbf{v}}_h}{\partial t} + f\mathbf{k} \times \hat{\mathbf{v}} = -g\nabla_h h - \frac{1}{\rho_0}\widehat{\nabla_h p^\star} + A_{Mh}\nabla_h^2 \hat{\mathbf{v}}_h + \frac{\boldsymbol{\tau}}{\rho_0 H} + \hat{\mathbf{F}}_c \; , \tag{1.12}$$

$$\frac{\partial h}{\partial t} = -H\nabla \cdot \hat{\mathbf{v}}_h \; , \tag{1.13}$$

where $p^\star = p^* - g\rho_0 h$. In this equation $\widehat{(..)} = 1/H \int_{-H}^{0} ..dz$. denotes the vertical averaging operator. The prime $(..)'$ is defined by the deviation from the vertical average. The equations for the baroclinic quantities become

$$\begin{aligned} \frac{\partial \mathbf{v}'_h}{\partial t} + f\mathbf{k} \times \mathbf{v}'_h &= -\frac{1}{\rho_0}\left(\nabla_h p^\star - \widehat{\nabla_h p^\star}\right) + A_{Mh}\nabla_h^2 \mathbf{v}'_h \\ &\quad + A_{Mv}\frac{\partial^2 \mathbf{v}'_h}{\partial z^2} - \frac{\boldsymbol{\tau}}{\rho_0 H} + \mathbf{F}'_c \; , \end{aligned} \tag{1.14}$$

$$\frac{\partial p^\star}{\partial z} = -g\rho^* \; , \tag{1.15}$$

$$\frac{\partial w}{\partial z} = -\nabla_h \cdot \mathbf{v}'_h \; , \tag{1.16}$$

$$\rho^* = \rho_0 \left(-\alpha_T T^* + \alpha_S S^*\right) \; , \tag{1.17}$$

$$\frac{\partial T^*}{\partial t} + \nabla_h \cdot (\mathbf{v}_h T^*) + \frac{\partial w T^*}{\partial z} = A_{Th}\nabla_h^2 T^* + A_{Tv}\frac{\partial^2 T^*}{\partial z^2} \; , \tag{1.18}$$

$$\frac{\partial S^*}{\partial t} + \nabla_h \cdot (\mathbf{v}_h S^*) + \frac{\partial w S^*}{\partial z} = A_{Th}\nabla_h^2 S^* + A_{Tv}\frac{\partial^2 S^*}{\partial z^2} \; . \tag{1.19}$$

For the spectral solution technique it is useful to replace the momentum equation by vorticity and divergence equations. Furthermore, the variables are nondimensionalized. The timescale is Ω^{-1}, the horizontal length scale a, the horizontal velocity scale Ωa, the vertical velocity scale ΩH, the height scale H, the pressure scale $g\rho_0 H$, the temperature scale $-1/\alpha_T$ and the salinity scale 1PSU. In the following all variables are understood as nondimensional variables.

Applying the operators $-\Omega^{-2}\nabla_h \cdot \mathbf{k}\times$ and $\Omega^{-2}\nabla_h \cdot$ gives the nondimensional vorticity and divergence equations, respectively

$$\frac{\partial \hat{\zeta}}{\partial t} = -2\mu\hat{D} - 2\hat{V} + A_{Mh}\left(\nabla_h^2\hat{\zeta} + 2\hat{\zeta}\right) - \nabla_h \cdot (\mathbf{k}\times\boldsymbol{\tau}) - \nabla_h \cdot \left(\mathbf{k}\times\hat{\mathbf{F}}_c\right) \ , \tag{1.20}$$

$$\begin{aligned}\frac{\partial \hat{D}}{\partial t} &= 2\mu\hat{\zeta} - 2\hat{U} + A_{Mh}\nabla_h^2\hat{D} - \frac{1}{\mathrm{Fr}^2}\nabla_h^2 h - \nabla_h^2\widehat{(p^\star)} \\ &\quad +\nabla_h \cdot \boldsymbol{\tau} + \nabla_h \cdot \hat{\mathbf{F}}_c \ ,\end{aligned} \tag{1.21}$$

$$\begin{aligned}\frac{\partial \zeta'}{\partial t} &= -2\mu D' - 2V' + A_{Mh}\left(\nabla_h^2\zeta' + 2\zeta'\right) + A_{Mv}\frac{\partial^2\zeta'}{\partial z^2} \\ &\quad +\nabla_h \cdot (\mathbf{k}\times\boldsymbol{\tau}) - \nabla_h \cdot (\mathbf{k}\times\mathbf{F}'_c) \ ,\end{aligned} \tag{1.22}$$

$$\begin{aligned}\frac{\partial D'}{\partial \tau} &= 2\mu\zeta' - 2U' + A_{Mh}\nabla_h^2 D' + A_{Mv}\frac{\partial^2 D'}{\partial z^2} \\ &\quad -\nabla_h^2\left(p^\star - \widehat{(p^\star)}\right) - \nabla_h \cdot \boldsymbol{\tau} + \nabla_h \cdot \mathbf{F}'_c \ .\end{aligned} \tag{1.23}$$

where $\mathrm{Fr} = \Omega a/\sqrt{gH}$ denotes the FROUDE number, $\zeta = -\nabla_h \cdot (\mathbf{k}\times\mathbf{v}_h)$ the vorticity, $D = \nabla_h \cdot \mathbf{v}_h$ the horizontal divergence, $U = u\cos\varphi$, $V = v\cos\varphi$ and $\mu = \sin\varphi$. The remaining equations become in nondimensional form

$$\frac{\partial h}{\partial t} = -\hat{D} \ , \tag{1.24}$$

$$\frac{\partial p^\star}{\partial z} = -\left(T^* + \alpha_S S^*\right) \ , \tag{1.25}$$

$$\frac{\partial w}{\partial z} = -D' \ , \tag{1.26}$$

$$\frac{\partial T^*}{\partial t} + \nabla_h \cdot (\mathbf{v}_h T^*) + \frac{\partial w T^*}{\partial z} = A_{Th} \nabla_h^2 T^* + A_{Tv} \frac{\partial^2 T^*}{\partial z^2} \ , \tag{1.27}$$

$$\frac{\partial S^*}{\partial t} + \nabla_h \cdot (\mathbf{v}_h S^*) + \frac{\partial w S^*}{\partial z} = A_{Th} \nabla_h^2 S^* + A_{Tv} \frac{\partial^2 S^*}{\partial z^2} \ . \tag{1.28}$$

2 Numerical solution technique

The model equations are solved with a semispectral and semi-implicit solution technique. In vertical direction discrete levels are introduced while the horizontal dependence is represented by spherical harmonic functions. The time integration scheme is semi-implicit by which all wave terms are stepped implicitly. The temperature and salinity advection will be calculated with the flux-form semi-Lagrangian (FFSL) transport algorithm of Lin and Rood (1996). It preserves volume integrated quantities, the positive definiteness of mass densities as well as constant fields.

2.1 Vertical discretisation

The ocean is divided into N layers and the middle of these layers represents the full level at which the variables except for vertical velocity are evaluated (see Figure 1). The vertical velocity is considered at the upper and lower boundaries of these layers (half levels) instead at the centres to obtain a higher accuracy for the vertical differencing scheme.

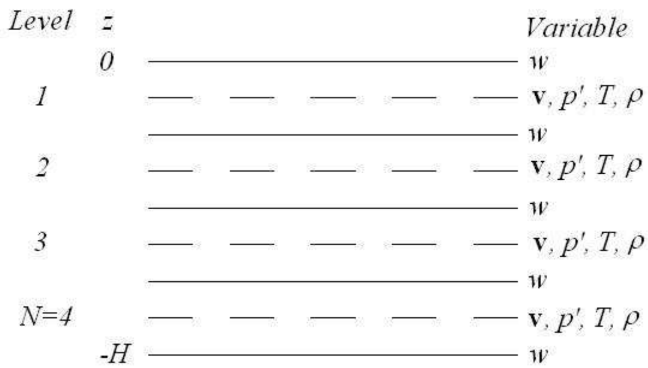

Figure 1: Arrangement of the vertical levels in SOM for $N = 4$.

The discrete level arrangement leads to the following approximations:

$$\frac{\partial w}{\partial z} \approx \frac{w_{j-1/2} - w_{j+1/2}}{z_{j-1/2} - z_{j+1/2}} \approx -\nabla_h \cdot \mathbf{v}_{hj} \ , \tag{2.1}$$

$$\frac{\partial p^\star}{\partial z} \approx \frac{p^\star_{j-1} - p^\star_j}{z_{j-1} - z_j} \approx -\frac{1}{2}\left(T^*_{j-1} + T^*_j\right) \ , \tag{2.2}$$

$$\frac{\partial^2 G}{\partial z^2} \approx \frac{(G_{j-1}-G_j)/(z_{j-1}-z_j)-(G_j-G_{j+1})/(z_j-z_{j+1})}{2(z_{j-1/2}-z_{j+1/2})} \quad (2.3)$$
$$\text{for} \quad G=T^* \ , \ G=S^* \ , \ G=\zeta' \text{ or } G=D' \ ,$$

$$\hat{p}^\star = \sum_{j=1}^{N} p_j^\star \left(z_{j-1/2}-z_{j+1/2}\right) \ . \quad (2.4)$$

In these equations the indices $j = 1, ..., N$ and $j - 1/2 = 1/2, ..., N + 1/2$ denote the full and half levels, respectively.

2.2 Spectral technique

All fields are projected onto a new base, namely, the so-called spherical harmonics $Y_n^m(\lambda, \mu)$. This gives for a field function $Q(\lambda, \mu, \tau)$:

$$Q(\lambda,\mu,t) = \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} Q_n^m(t) Y_n^m(\lambda,\mu) \ . \quad (2.5)$$

The spherical harmonics are defined by

$$Y_n^m(\lambda,\mu) = \sqrt{\frac{2n+1}{2}\frac{(n-m)!}{(n+m)!}}\frac{(1-\mu^2)^{\frac{m}{2}}}{2^n n!}\frac{\partial^{m+n}}{\partial\mu^{m+n}}\left[(\mu^2-1)^n\right]e^{im\lambda} \ . \quad (2.6)$$

Since Q must be a real function we have the constraint $Q_n^m = Q_n^{-m*}$. Therefore, it is sufficient to calculate only the coefficients with $m \geq 0$ and the following representation is equivalent to (2.6):

$$Q(\lambda,\mu,\tau) = \sum_{n=1}^{\infty} Q_n^0(\tau)Y_n^0(\mu) + 2\sum_{m=1}^{\infty}\sum_{n=m}^{\infty} \mathrm{Re}\left(Q_n^m(\tau)Y_n^m(\lambda,\mu)\right) \ . \quad (2.7)$$

The spherical harmonics form an orthogonal base with respect to the scalar product

$$\langle Q_1|Q_2\rangle := \frac{1}{2\pi}\int_{-1}^{1}\int_{0}^{2\pi} Q_1^*(\lambda,\mu)Q_2(\lambda,\mu)d\lambda d\mu \ . \quad (2.8)$$

Therefore:

$$\left\langle Y_{n_1}^{m_1}|Y_{n_2}^{m_2}\right\rangle = \delta_{m_1,m_2}\delta_{n_1,n_2} \ . \quad (2.9)$$

Furthermore, the spherical harmonics are eigenfunctions of the nondimensional LAPLACE-operator ∇_h^2

$$\nabla_h^2 Y_n^m = -n(n+1) Y_n^m \; . \tag{2.10}$$

Further useful relations are:

$$\frac{\partial Y_n^m}{\partial \lambda} = im Y_n^m \; , \tag{2.11}$$

$$\mu Y_n^m = d_{m,n+1} Y_{n+1}^m + d_{m,n} Y_{n-1}^m \; , \tag{2.12}$$

$$(1-\mu^2)\frac{\partial}{\partial \mu} Y_n^m = n d_{m,n+1} Y_{n+1}^m - (n+1) d_{m,n} Y_{n-1}^m \; , \text{ where } d_{m,n} = \sqrt{\frac{n^2 - m^2}{4n^2 - 1}} \; . \tag{2.13}$$

With (2.11) - (2.13) the governing model equations can be converted into component form after applying the scalar product $< Y_n^m|.. >$:

$$\frac{d\hat{\zeta}_n^m}{dt} + 2(d_{m,n}\hat{D}_{n-1}^m + d_{m,n+1}\hat{D}_{n+1}^m + \hat{V}_n^m) + A_{Mh}\left(n^2 + n - 2\right)\hat{\zeta}_n^m - \left(C_{\hat{\mathbf{F}}_c}\right)_n^m = C_{\hat{\boldsymbol{\tau}}}{}_n^m \; , \tag{2.14}$$

$$\frac{d\hat{D}_n^m}{dt} - 2(d_{m,n}\hat{\zeta}_{n-1}^m + d_{m,n+1}\hat{\zeta}_{n+1}^m - \hat{U}_n^m) + A_{Mh} n(n+1) D_n^m - \left(D_{\hat{\mathbf{F}}_c}\right)_n^m - \frac{n(n+1)}{\mathrm{Fr}^2} h_n^m = D_{\hat{\boldsymbol{\tau}}}{}_n^m + n(n+1)\widehat{(p^\star)} \; , \tag{2.15}$$

$$\frac{dh_n^m}{dt} = -\hat{D}_n^m \; , \tag{2.16}$$

$$\left[\frac{d\hat{\zeta}'{}_n^m}{dt} + 2(d_{m,n}D'{}_{n-1}^m + d_{m,n+1}D'{}_{n+1}^m + V'{}_n^m) + A_{Mh}\left(n^2 + n - 2\right)\hat{\zeta}'{}_n^m - \left(C_{\mathbf{F}'_c}\right)_n^m\right]_j = A_{Mv}\frac{\frac{\zeta'{}_{n\,(j-1)}^m - \zeta'{}_{n\,j}^m}{z_{j-1} - z_j} - \frac{\zeta'{}_{n\,j}^m - \zeta'{}_{n\,(j+1)}^m}{z_j - z_{j+1}}}{z_j - z_{j+1}} + C_{\boldsymbol{\tau}'}{}_{n\,j}^m \; , \tag{2.17}$$

$$\left[\frac{dD'^{\,m}_n}{dt} - 2(d_{m,n}\zeta'^{\,m}_n + d_{m,n+1}\zeta'^{\,m}_n - U'^{\,m}_n) + A_{Mh}n(n+1)D'^{\,m}_n - \left(D_{\mathbf{F}'_c}\right)^m_n\right]_j$$
$$= A_{Mv}\frac{\frac{D'^{\,m}_{n\ (j-1)}-D'^{\,m}_{n\ j}}{z_{j-1}-z_j} - \frac{D'^{\,m}_{n\ j}-D'^{\,m}_{n\ (j+1)}}{z_j-z_{j+1}}}{2(z_{j-1/2}-z_{j+1/2})} + D_{\boldsymbol{\mathcal{T}}'}{}^m_{n\ j} + n(n+1)(p^{\star\prime})^m_{n\ j}\ , \tag{2.18}$$

$$\frac{p^{\star m}_{n\ (j-1)} - p^{\star m}_{n\ j}}{z_{j-1}-z_j} = -\frac{1}{2}\left(T^{*m}_{n\ (j-1)} + T^{*m}_{n\ j}\right)\ , \tag{2.19}$$

where $(C_{\mathbf{A}})^m_n$ $((D_{\mathbf{A}})^m_n)$ denotes the spectral coefficient of the curl (divergence) of the vector $\mathbf{A}$. The linear terms on the left hand side of Eqs. (2.14)-(2.17) are integrated with an implicit time scheme. For this purpose a time-independent matrix has to be inverted prior to the model simulation. The remaining terms are stepped with the Leap-frog scheme. The time step of the model can be chosen in the order of a day since the explicitly treated tendencies describe slow processes in the ocean. It is, furthermore, possible to run SOM as a planetary geostrophic ocean model. Then, the time tendencies in the momentum equation and surface elevation equation are set to zero.

For noise reduction a ROBERT/ASSELIN-Filter is applied at every timestep to a spectral coefficient Q^m_n as follows:

$$Q^m_n(t) = (1-2\gamma)Q^m_n(t) + \gamma[Q^m_n(t+\delta t) + Q^m_n(t-\delta t)]\ , \tag{2.20}$$

where δt denotes the time step length. In SOM γ takes the value 0.02 by default.

The temperature and salinity equations are solved in gridpoint space since the semi-Lagrangian transport scheme operates in gridpoint space only. The velocity fields for this scheme are determined from vorticity and divergence using the Helmholtz-theorem

$$\zeta = \nabla^2_h\psi,\ D = \nabla^2_h\chi\ ,\ \mathbf{v}_h = \mathbf{k}\times\nabla_h\psi + \nabla_h\chi\ , \tag{2.21}$$

where ψ denotes the streamfunction and χ the velocity potential. After transformation in gridpoint space the fields are used for the semi-Lagrangian transport solver. The solver determines the vertical velocity w from the continuity equation (2.1). Before application the velocity is exactly set to zero above land regions. Before transforming to spectral space the gridpoint field of the temperature and salinity will be embedded into another fields which suffer strong horizontal diffusion over land regions. With this method the land sea contrast will be smoothed and the spectral representation of the density fields becomes better.

2.3 Parameterization of convection

It is possible that the stratification becomes statically unstable locally. In such a case convection must be parameterized suitably. This will be done by convective adjustment in gridpoint space. A vertical exchange of water masses takes place if a box has a smaller density than the box below. Then, the new temperatures T^{*N} of the boxes with level j and level $j+1$ become

$$T_j^{*N} = T^*_{(j+1)} \ , \ T^{*N}_{(j+1)} = T^*_j \frac{\delta z_j}{\delta z_{j+1}} + T^*_{N\ (j+1)} \frac{\delta z_{j+1} - \delta z_j}{\delta z_{j+1}} \ , \tag{2.22}$$

where $\delta z_j = z_{j-1/2} - z_{j+1/2}$. Salinity S^* is treated in the same way.

2.4 Hyperdiffusion

Optionally, additional smoothing of the fields can be oreached by the inclusion of hyperdiffusion. Hyperdiffusion efficiently damps the high wavenumber pattern by the multiple application of the LAPLACE operator. Therefore, hyperdiffusion is included by adding the following terms

$$\mathcal{H}^m_{G\ n} = K\left[-n(n+1)\right]^{n_h} G^m_n \tag{2.23}$$

to the prognostic spectral equation for the spectral coefficient G^m_n (vorticity and divergence). n_h denotes the order of hyperdiffusion. Without additional terms the corresponding vorticit or divergence component is damped with the dimensional e-folding timescale

$$\tau_H = \left\{\Omega K[n(n+1)]^{n_h}\right\}^{-1} \ . \tag{2.24}$$

In the model τ_H at the wavenumber where the expansion is truncated will be prescribed. Therefore, the coefficient K becomes:

$$K = \left\{\Omega \tau_H [M(M+1)]^{n_h}\right\}^{-1} \ . \tag{2.25}$$

where M denotes the truncation wavenumber.

3 Coupling to the PlanetSimulator

In the usual set-up SOM is coupled to the PlanetSimulator (Fraedrich et al. 2005). For this purpose the mixed layer model of the PlanetSimulator shares the uppermost layer with SOM. The PlanetSimulator predicts the temperature changes due to radiative and air-to sea heat fluxes F_A while SOM calculates the tendencies due to advection and diffusion. The fresh water flux F_S will be calculated by the PlanetSimulator and it enters SOM in the salinity equation for the mixed layer according to boundary condition (1.10). Furthermore, the sea ice model of the PlanetSimulator is applied and the predicted sea ice cover is used in SOM. SOM interprets sea ice as a layer where no momentum can be transferred into the ocean. Therefore, the wind stress becomes zero in regions with sea ice.

4 User guide

To run the PlanetSimulator with SOM it is necessary to set NSOM=1, NLSG=0 and NOCEAN=1 in the *ocean_namelist*. The SOM-namelist *somnamelist* contains the following parameters

Parameter	Default	Meaning
NAFTER	12	Parameter to determine that output should be written after intervals of NAFTER timesteps
NCOEFF	0	Number of spectral modes to be written out in the diagnostics subroutine prispsom
NDEL	6	Order of hyperdiffusion
NDIAG	12	Parameter to determine that ASCII diagnostics are written after intervals of NDIAG timesteps
NKITS	3	Number of initial timesteps
NOUTPUT	1	Global switch for output on (1) or off (0)
NTSPD	1	Number of timesteps per day
NCU	0	Check unit (for debug output only)
TDISS	0.25 d	Hyper diffusion time scale
ROTSPD	1.0	Rotations per day (should be set as in PlaSim)
LRESTART	.false.	Switch for restart
AMH	10^6 m^2/s	Horizontal momentum exchange coefficient
ATH	2000 m^2/s	Horizontal temperature exchange coefficient
AMV	0 m^2/s	Vertical momentum exchange coefficient
ATV	5×10^{-5} m^2/s	Vertical temperature exchange coefficient
LSALT	.true.	Switch for inclusion of salinity
LLSG	.false.	Switch for the planetary geostrophic approximation
LBARO	.false.	Switch for barotropic integration
LLID	.false.	Switch for introducing a rigid lid
LMATRIX	.false.	Switch for matrix determination

The following parameters can only be modified in the model code.

Parameter	Default	Meaning
GA	9.81 m/s^2	Gravity acceleration g
PLARAD	6.371×10^6m	Earth radius a
WW	0.7292×10^{-4}s^{-1}	Angular velocity Ω of solid Earth
RHO0	1030kg/m^3	Mean density of sea water ρ_0
CPW	4180 J(kgK)$^{-1}$	Heat capacity of sea water c_w
ALPHAT	1.6×10^{-4}K^{-1}	Thermal expansion coefficient α_T for sea water
ALPHAS	7.6×10^{-4}K^{-1}	Salinity expansion coefficient α_S for sea water
S0	35 PSU	Mean salinity S_0
HS	4000m	Mean depth of ocean H

To ensure consistency, the parameters GA, PLARAD, WW, RHO0, CPW should be identical to the corresponding parameters in the PlanetSimulator if a coupled run will be performed.

SOM inverts the matrix containing the implicit linear tendencies prior to the model run. The matrix is written into the file *som_matrix*. After the first restart the matrix will not be determined again. Currently, SOM assumes an annual cycle of 360 days.

SOM writes output into the file *som_output*. It contains spectral coefficients of i) geopotential of sea surface elevation (code 152, not implemented), ii) divergence (code 155), iii) relative vorticity (code 138), iv) temperature (code 130), and land sea mask (code 272). The pumaburner can process the file *som_output* for further diagnostics.

Gridpoint fields associated with the semi-Lagrangian advection scheme are stored in the service format file *som_outgr.srv*. It contains gridpoint fields of
- temperature [K](code 130),
- zonal velocity [m/s] (code 131),
- meridional velocity [m/s] (code 132),
- vertical velocity [m/s] (code 137),
- salinity [PSU] (code 279),
- density tendency due to convective mixing [kg/m^3/s] (code 280),
- barotropic mass streamfunction [Sv] (code 148),
- geopotential height elevation of sea surface [m] (code 156),
- zonal wind stress [N/m^2](code 180),
- meridional wind stress [N/m^2](code 181),
- air to sea heat flux [W/m^2] (code 263),
- net fresh water flux [m/s] (code 264),
- fresh water flux due to surface runoff [m/s] (code 160),
- sea ice thickness [m] (code 211),
- land sea mask (code 172).

The standard SOM version adopts 16 height levels which are given by

Number	Half level	full level	layer thickness
0	0m	-	-
1	-50.0m	25.0m	50.0m
2	-94.0m	72.0m	44.0m
3	-160.5m	127.2m	66.5m
4	-244.3m	202.4m	83.8m
5	-349.4m	296.8m	105.1m
6	-480.3m	414.8m	130.9m
7	-642.0m	561.1m	161.7m
8	-839.9m	741.0m	197.9m
9	-1079.3m	959.6m	239.4m
10	-1364.6m	1222.0m	285.3m
11	-1698.8m	1531.7m	334.2m
12	-2082.6m	1890.7m	383.7m
13	-2513.1m	2297.8m	430.5m
14	-2983.8m	2748.4m	470.7m
15	-3484.0m	3233.9m	500.3m
16	-4000.0m	3742.0m	516.0m

It is ensured that the mixed-layer depth of PlaSim is identical to the thickness of the uppermost layer of SOM. An interface to run SOM standalone will be developed in the near future.

References

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